

SECOND EDITION

INTRODUCTORY **BIOSTATISTICS**

Chap T. Le • Lynn E. Eberly



WILEY

INTRODUCTORY BIostatISTICS

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Second Edition

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To my wife, Minhha, and my daughters, Mina and Jenna with love

C.T.L.

*To my husband, Andy, and my sons, Evan, Jason, and Colin, with love;
you bring joy to my life*

L.E.E.

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PREFACE TO THE SECOND EDITION

This second edition of the book adds several new features:

- An expanded treatment of one-way ANOVA including multiple testing procedures;
- A new chapter on two-way, three-way, and higher level ANOVAs, including both fixed, random, and mixed effects ANOVAs;
- A substantially revised chapter on regression;
- A new chapter on models for repeated measurements using linear mixed models and generalized estimating equations;
- Examples worked throughout the book in R in addition to SAS software;
- Additional end of chapter exercises in several chapters.

These features have been added with the help of a new second author. As in the first edition, data sets used in the in-chapter examples and end of chapter exercises are largely based on real studies on which we collaborated. The very large data tables referred to throughout this book are too large for inclusion in the printed text; they are available at www.wiley.com/go/Le/Biostatistics.

We thank previous users of the book for feedback on the first edition, which led to many of the improvements in this second edition. We also thank Megan Schlick, Division of Biostatistics at the University of Minnesota, for her assistance with preparation of several files and the index for this edition.

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September 2015

PREFACE TO THE FIRST EDITION

A course in introductory biostatistics is often required for professional students in public health, dentistry, nursing, and medicine, and for graduate students in nursing and other biomedical sciences, a requirement that is often considered a roadblock, causing anxiety in many quarters. These feelings are expressed in many ways and in many different settings, but all lead to the same conclusion: that students need help, in the form of a user-friendly and real data-based text, in order to provide enough motivation to learn a subject that is perceived to be difficult and dry. This introductory text is written for professionals and beginning graduate students in human health disciplines who need help to pass and benefit from the basic biostatistics requirement of a one-term course or a full-year sequence of two courses. Our main objective is to avoid the perception that statistics is just a series of formulas that students need to “get over with,” but to present it as a way of thinking – thinking about ways to gather and analyze data so as to benefit from taking the required course. There is no better way to do that than to base a book on real data, so many real data sets in various fields are provided in the form of examples and exercises as aids to learning how to use statistical procedures, still the nuts and bolts of elementary applied statistics.

The first five chapters start slowly in a user-friendly style to nurture interest and motivate learning. Sections called “Brief Notes on the Fundamentals” are added here and there to gradually strengthen the background and the concepts. Then the pace is picked up in the remaining seven chapters to make sure that those who take a full-year sequence of two courses learn enough of the nuts and bolts of the subject. Our basic strategy is that most students would need only one course, which would end at about the middle of Chapter 9, after covering simple linear regression; instructors may add a few sections of Chapter 14. For students who take only one course, other chapters would serve as references to supplement class discussions as well as for

their future needs. A subgroup of students with a stronger background in mathematics would go on to a second course, and with the help of the brief notes on the fundamentals would be able to handle the remaining chapters. A special feature of the book is the sections “Notes on Computations” at the end of most chapters. These notes cover the uses of Microsoft’s Excel, but samples of SAS computer programs are also included at the end of many examples, especially the advanced topics in the last several chapters.

The way of thinking called *statistics* has become important to all professionals, not only those in science or business, but also caring people who want to help to make the world a better place. But what is biostatistics, and what can it do? There are popular definitions and perceptions of statistics. We see “vital statistics” in the newspaper: announcements of life events such as births, marriages, and deaths. Motorists are warned to drive carefully, to avoid “becoming a statistic.” Public use of the word is widely varied, most often indicating lists of numbers, or data. We have also heard people use the word *data* to describe a verbal report, a believable anecdote. For this book, especially in the first few chapters, we do not emphasize statistics as things, but instead, offer an active concept of “doing statistics.” The doing of statistics is a way of thinking about numbers (collection, analysis, presentation), with emphasis on relating their interpretation and meaning to the manner in which they are collected. Formulas are only a part of that thinking, simply tools of the trade; they are needed but not as the only things one needs to know.

To illustrate statistics as a way of thinking, let us begin with a familiar scenario: criminal court procedures. A crime has been discovered and a suspect has been identified. After a police investigation to collect evidence against the suspect, a prosecutor presents summarized evidence to a jury. The jurors are given the rules regarding convicting beyond a reasonable doubt and about a unanimous decision, and then they debate. After the debate, the jurors vote and a verdict is reached: guilty or not guilty. Why do we need to have this time-consuming, cost-consuming process of trial by jury? One reason is that the truth is often unknown, at least uncertain. Perhaps only the suspect knows but he or she does not talk. It is uncertain because of variability (every case is different) and because of possibly incomplete information. Trial by jury is the way our society deals with uncertainties; its goal is to minimize mistakes.

How does society deal with uncertainties? We go through a process called *trial by jury*, consisting of these steps: (1) we form an assumption or hypothesis (that every person is innocent until proved guilty), (2) we gather data (evidence against the suspect), and (3) we decide whether the hypothesis should be rejected (guilty) or should not be rejected (not guilty). With such a well-established procedure, sometimes we do well, sometimes we do not. Basically, a successful trial should consist of these elements: (1) a probable cause (with a crime and a suspect), (2) a thorough investigation by police, (3) an efficient presentation by a prosecutor, and (4) a fair and impartial jury.

In the context of a trial by jury, let us consider a few specific examples: (1) the *crime* is lung cancer and the *suspect* is cigarette smoking, or (2) the *crime* is leukemia and the *suspect* is pesticides, or (3) the *crime* is breast cancer and the *suspect* is a defective gene. The process is now called *research* and the tool to carry out that research is biostatistics. In a simple way, biostatistics serves as the biomedical

version of the trial by jury process. It is the *science of dealing with uncertainties using incomplete information*. Yes, even science is uncertain; scientists arrive at different conclusions in many different areas at different times; many studies are inconclusive (hung jury). The reasons for uncertainties remain the same. Nature is complex and full of unexplained biological variability. But most important, we always have to deal with incomplete information. It is often not practical to study an entire population; we have to rely on information gained from a *sample*.

How does science deal with uncertainties? We learn how society deals with uncertainties; we go through a process called *biostatistics*, consisting of these steps: (1) we form an assumption or hypothesis (from the research question), (2) we gather data (from clinical trials, surveys, medical record abstractions), and (3) we make decision(s) (by doing statistical analysis/inference; a guilty verdict is referred to as *statistical significance*). Basically, a successful research should consist of these elements: (1) a good research question (with well-defined objectives and endpoints), (2) a thorough investigation (by experiments or surveys), (3) an efficient presentation of data (organizing data, summarizing, and presenting data: an area called *descriptive statistics*), and (4) proper statistical inference. This book is a problem-based introduction to the last three elements; together they form a field called *biostatistics*. The coverage is rather brief on data collection but very extensive on descriptive statistics (Chapters 1, 2), especially on methods of statistical inference (Chapters 4–12). Chapter 3, on probability and probability models, serves as the link between the descriptive and inferential parts. Notes on computations and samples of SAS computer programs are incorporated throughout the book. About 60% of the material in the first eight chapters overlaps with chapters from *Health and Numbers: A Problems-Based Introduction to Biostatistics* (another book by Wiley), but new topics have been added and others rewritten at a somewhat higher level. In general, compared to *Health and Numbers*, this book is aimed at a different audience – those who need a whole year of statistics and who are more mathematically prepared for advanced algebra and precalculus subjects.

I would like to express my sincere appreciation to colleagues, teaching assistants, and many generations of students for their help and feedback. I have learned very much from my former students, I hope that some of what they have taught me is reflected well in many sections of this book. Finally, my family bore patiently the pressures caused by my long-term commitment to the book; to my wife and daughters, I am always most grateful.

Chap T. Le
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ABOUT THE COMPANION WEBSITE

This book is accompanied by a companion website:

www.wiley.com/go/Le/Biostatistics

The website includes:

- Electronic copy of the larger data sets used in Examples and Exercises

1

DESCRIPTIVE METHODS FOR CATEGORICAL DATA

Most introductory textbooks in statistics and biostatistics start with methods for summarizing and presenting continuous data. We have decided, however, to adopt a different starting point because our focused areas are in the biomedical sciences, and health decisions are frequently based on proportions, ratios, or rates. In this first chapter we will see how these concepts appeal to common sense, and learn their meaning and uses.

1.1 PROPORTIONS

Many outcomes can be classified as belonging to one of two possible categories: presence and absence, nonwhite and white, male and female, improved and nonimproved. Of course, one of these two categories is usually identified as of primary interest: for example, presence in the presence and absence classification, nonwhite in the white and nonwhite classification. We can, in general, relabel the two outcome categories as positive (+) and negative (−). An outcome is *positive* if the primary category is observed and is *negative* if the other category is observed.

It is obvious that, in the summary to characterize observations made on a group of people, the number x of positive outcomes is not sufficient; the group size n , or total number of observations, should also be recorded. The number x tells us very little and becomes meaningful only after adjusting for the size n of the group; in other words, the two figures x and n are often combined into a *statistic*, called a *proportion*:

$$p = \frac{x}{n}.$$

The term *statistic* means a summarized quantity from observed data. Clearly, $0 \leq p \leq 1$. This proportion p is sometimes expressed as a percentage and is calculated as follows:

$$\text{percentage}(\%) = \frac{x}{n}(100).$$

Example 1.1

A study published by the Urban Coalition of Minneapolis and the University of Minnesota Adolescent Health Program surveyed 12915 students in grades 7–12 in Minneapolis and St. Paul public schools. The report stated that minority students, about one-third of the group, were much less likely to have had a recent routine physical checkup. Among Asian students, 25.4% said that they had not seen a doctor or a dentist in the last two years, followed by 17.7% of Native Americans, 16.1% of blacks, and 10% of Hispanics. Among whites, it was 6.5%.

Proportion is a number used to describe a group of people according to a *dichotomous*, or *binary, characteristic* under investigation. It is noted that characteristics with multiple categories can have a proportion calculated per category, or can be dichotomized by pooling some categories to form a new one, and the concept of proportion applies. The following are a few illustrations of the use of proportions in the health sciences.

1.1.1 Comparative Studies

Comparative studies are intended to show possible differences between two or more groups; Example 1.1 is such a typical comparative study. The survey cited in Example 1.1 also provided the following figures concerning boys in the group who use tobacco at least weekly. Among Asians, it was 9.7%, followed by 11.6% of blacks, 20.6% of Hispanics, 25.4% of whites, and 38.3% of Native Americans.

In addition to surveys that are cross-sectional, as seen in Example 1.1, data for comparative studies may come from different sources; the two fundamental designs being retrospective and prospective. *Retrospective studies* gather past data from selected cases and controls to determine differences, if any, in *exposure* to a suspected *risk factor*. These are commonly referred to as *case-control studies*; each such study is focused on a particular disease. In a typical case-control study, cases of a specific disease are ascertained as they arise from population-based registers or lists of hospital admissions, and controls are sampled either as disease-free persons from the population at risk or as hospitalized patients having a diagnosis other than the one under study. The advantages of a retrospective study are that it is economical and provides answers to research questions relatively quickly because the cases are already available. Major limitations are due to the inaccuracy of the exposure histories and uncertainty about the appropriateness of the control sample; these problems sometimes hinder retrospective studies and make them less preferred than prospective studies. The following is an example of a retrospective study in the field of occupational health.

Example 1.2

A case–control study was undertaken to identify reasons for the exceptionally high rate of lung cancer among male residents of coastal Georgia. Cases were identified from these sources:

1. Diagnoses since 1970 at the single large hospital in Brunswick;
2. Diagnoses during 1975–1976 at three major hospitals in Savannah;
3. Death certificates for the period 1970–1974 in the area.

Controls were selected from admissions to the four hospitals and from death certificates in the same period for diagnoses other than lung cancer, bladder cancer, or chronic lung cancer. Data are tabulated separately for smokers and nonsmokers in Table 1.1. The exposure under investigation, “shipbuilding,” refers to employment in shipyards during World War II. By using a separate tabulation, with the first half of the table for nonsmokers and the second half for smokers, we treat *smoking* as a potential confounder. A *confounder* is a factor, an exposure by itself, not under investigation but related to the disease (in this case, lung cancer) and the exposure (shipbuilding); previous studies have linked smoking to lung cancer, and construction workers are more likely to be smokers. The term *exposure* is used here to emphasize that employment in shipyards is a suspected *risk* factor; however, the term is also used in studies where the factor under investigation has beneficial effects.

In an examination of the smokers in the data set in Example 1.2, the numbers of people employed in shipyards, 84 and 45, tell us little because the sizes of the two groups, cases and controls, are different. Adjusting these absolute numbers for the group sizes (397 cases and 315 controls), we have:

1. For the smoking controls,

$$\begin{aligned} \text{proportion with exposure} &= \frac{45}{315} \\ &= 0.143 \text{ or } 14.3\%. \end{aligned}$$

2. For the smoking cases,

$$\begin{aligned} \text{proportion with exposure} &= \frac{84}{397} \\ &= 0.212 \text{ or } 21.2\%. \end{aligned}$$

TABLE 1.1

Smoking	Shipbuilding	Cases	Controls
No	Yes	11	35
	No	50	203
Yes	Yes	84	45
	No	313	270

The results reveal different exposure histories: the proportion in shipbuilding among cases was higher than that among controls. It is *not* in any way conclusive proof, but it is a good *clue*, indicating a possible *relationship* between the disease (lung cancer) and the exposure (shipbuilding).

Similar examination of the data for nonsmokers shows that, by taking into consideration the numbers of cases and controls, we have the following figures for shipbuilding employment:

1. For the non-smoking controls,

$$\begin{aligned}\text{proportion with exposure} &= \frac{35}{238} \\ &= 0.147 \text{ or } 14.7\%.\end{aligned}$$

2. For the non-smoking cases,

$$\begin{aligned}\text{proportion with exposure} &= \frac{11}{61} \\ &= 0.180 \text{ or } 18.0\%.\end{aligned}$$

The results for non-smokers also reveal different exposure histories: the proportion in shipbuilding among cases was again higher than that among controls.

The analyses above also show that the case-control difference in the proportions with the exposure among smokers, that is,

$$21.2 - 14.3 = 6.9\%,$$

is different from the case-control difference in the proportions with the exposure among nonsmokers, which is:

$$18.0 - 14.7 = 3.3\%.$$

The differences, 6.9% and 3.3%, are *measures* of the strength of the relationship between the disease and the exposure, one for each of the two strata: the two groups of smokers and nonsmokers, respectively. The calculation above shows that the possible effects of employment in shipyards (as a suspected risk factor) are different for smokers and nonsmokers. This difference of differences, if confirmed, is called a *three-term interaction* or *effect modification*, where smoking alters the effect of employment in shipyards as a risk for lung cancer. In that case, *smoking* is not only a confounder, it is an *effect modifier*, which modifies the effects of shipbuilding (on the possibility of having lung cancer).

Another illustration is provided in the following example concerning glaucomatous blindness.

TABLE 1.2

	Population	Cases	Cases per 100 000
White	32 930 233	2832	8.6
Nonwhite	3 933 333	3227	82.0

Example 1.3

Counts of persons registered blind from glaucoma are listed in Table 1.2.

For these *disease registry data*, direct calculation of a proportion results in a very tiny fraction, that is, the number of cases of the disease per person at risk. For convenience, in Table 1.2, this is multiplied by 100 000, and hence the result expresses the number of cases per 100 000 people. This data set also provides an example of the use of proportions as disease *prevalence*, which is defined as:

$$\text{prevalence} = \frac{\text{number of diseased persons at the time of investigation}}{\text{total number of persons examined}}.$$

Disease prevalence and related concepts are discussed in more detail in Section 1.2.2.

For blindness from glaucoma, calculations in Example 1.3 reveal a striking difference between the races: The blindness prevalence among nonwhites was over eight times that among whites. The number “100 000” was selected arbitrarily; any power of 10 would be suitable so as to obtain a result between 1 and 100, sometimes between 1 and 1000; it is easier to state the result “82 cases per 100 000” than to say that the prevalence is 0.00082.

1.1.2 Screening Tests

Other uses of proportions can be found in the evaluation of *screening tests* or *diagnostic procedures*. Following these procedures, using clinical observations or laboratory techniques, people are classified as healthy or as falling into one of a number of disease categories. Such tests are important in medicine and epidemiologic studies and may form the basis of early interventions. Almost all such tests are imperfect, in the sense that healthy persons will occasionally be classified wrongly as being ill, while some people who are really ill may fail to be detected. That is, *misclassification* is unavoidable. Suppose that each person in a large population can be classified as truly positive or negative for a particular disease; this true diagnosis may be based on more refined methods than are used in the test, or it may be based on evidence that emerges after the passage of time (e.g., at autopsy). For each class of people, diseased and healthy, the test is applied, with the results depicted in Figure 1.1.

The two proportions fundamental to evaluating diagnostic procedures are sensitivity and specificity. *Sensitivity* is the proportion of diseased people detected as positive by the test:

$$\text{sensitivity} = \frac{\text{number of diseased persons who test positive}}{\text{total number of diseased persons}}.$$

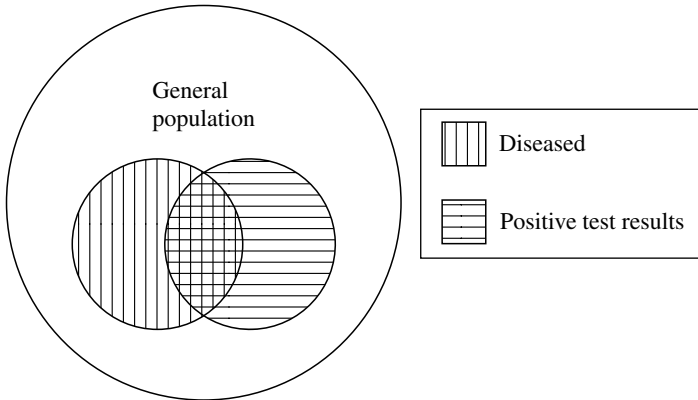


FIGURE 1.1 Graphical display of a screening test.

The corresponding errors are *false negatives*. *Specificity* is the proportion of healthy people detected as negative by the test:

$$\text{specificity} = \frac{\text{number of healthy persons who test negative}}{\text{total number of healthy persons}}.$$

The corresponding errors are *false positives*.

Clearly, it is desirable that a test or screening procedure be highly sensitive and highly specific. However, the two types of errors go in opposite directions; for example, an effort to increase sensitivity may lead to more false positives, and vice versa.

Example 1.4

A cytological test was undertaken to screen women for cervical cancer. Consider a group of 24 103 women consisting of 379 women whose cervixes are abnormal (to an extent sufficient to justify concern with respect to possible cancer) and 23 724 women whose cervixes are acceptably healthy. A test was applied and results are tabulated in Table 1.3. (This study was performed with a rather old test and is used here only for illustration.)

TABLE 1.3

True	Test		Total
	-	+	
-	23 362	362	23 724
+	225	154	379

The calculations

$$\begin{aligned}\text{sensitivity} &= \frac{154}{379} \\ &= 0.406 \text{ or } 40.6\% \\ \text{specificity} &= \frac{23\,362}{23\,724} \\ &= 0.985 \text{ or } 98.5\%\end{aligned}$$

show that the test is highly specific (98.5%) but not very sensitive (40.6%); among the 379 women with the disease, more than half (59.4%) had false negatives. The implications of the use of this test are:

1. If a woman without cervical cancer is tested, the result would almost surely be negative, *but*
2. If a woman with cervical cancer is tested, the chance is that the disease would go undetected because 59.4% of these cases would result in false negatives.

Finally, it is important to note that throughout this section, proportions have been defined so that both the numerator and the denominator are counts or frequencies, and the numerator corresponds to a subgroup of the larger group involved in the denominator, resulting in a number between 0 and 1 (or between 0 and 100%). It is straightforward to generalize this concept for use with characteristics having more than two outcome categories; for each category we can define a proportion, and these category-specific proportions add up to 1 (or 100%).

Example 1.5

An examination of the 668 children reported living in crack/cocaine households shows 70% blacks, followed by 18% whites, 8% Native Americans, and 4% other or unknown.

1.1.3 Displaying Proportions

Perhaps the most effective and most convenient way of presenting data, especially discrete data, is through the use of graphs. Graphs convey the information, the general patterns in a set of data, at a single glance. Therefore, graphs are often easier to read than tables; the most informative graphs are simple and self-explanatory. Of course, to achieve that objective, graphs should be constructed carefully. Like tables, they should be clearly labeled and units of measurement and/or magnitude of quantities should be included. Remember that graphs must tell their own story; they should be complete in themselves and require little or no additional explanation.

Bar Charts Bar charts are a very popular type of graph used to display several proportions for quick comparison. In applications suitable for bar charts, there are several groups and we investigate one binary characteristic. In a bar chart, the various

groups are represented along the horizontal axis; they may be arranged alphabetically, by the size of their proportions, or on some other rational basis. A vertical bar is drawn above each group such that the height of the bar is the proportion associated with that group. The bars should be of equal width and should be separated from one another so as not to imply continuity.

Example 1.6

We can present the data set on children without a recent physical checkup (Example 1.1) by a bar chart, as shown in Figure 1.2.

Pie Charts Pie charts are another popular type of graph. In applications suitable for pie charts, there is only one group but we want to decompose it into several categories. A pie chart consists of a circle; the circle is divided into wedges that correspond to the magnitude of the proportions for various categories. A pie chart shows the differences between the sizes of various categories or subgroups as a decomposition of the total. It is suitable, for example, for use in presenting a budget, where we can easily see the difference between United States expenditures on health care and defense. In other words, a bar chart is a suitable graphic device when we have several groups, each associated with a different proportion; whereas a pie chart is more suitable when we have one group that is divided into several categories. The proportions of various categories in a pie chart should add up to 100%. Like bar charts, the categories in a pie chart are usually arranged by the size of the proportions. They may also be arranged alphabetically or on some other rational basis.

Example 1.7

We can present the data set on children living in crack households (Example 1.5) by a pie chart as shown in Figure 1.3.

Another example of the pie chart's use is for presenting the proportions of deaths due to different causes.

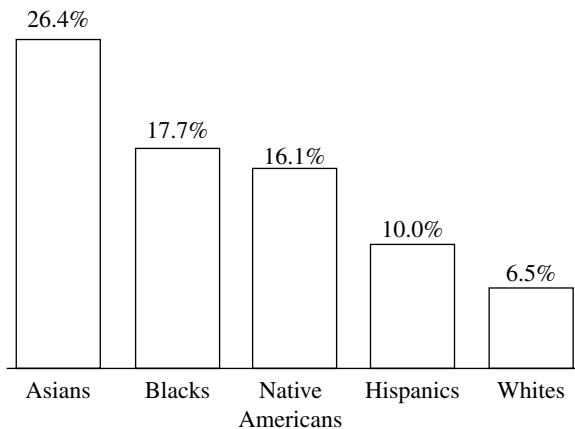


FIGURE 1.2 Children without a recent physical checkup.

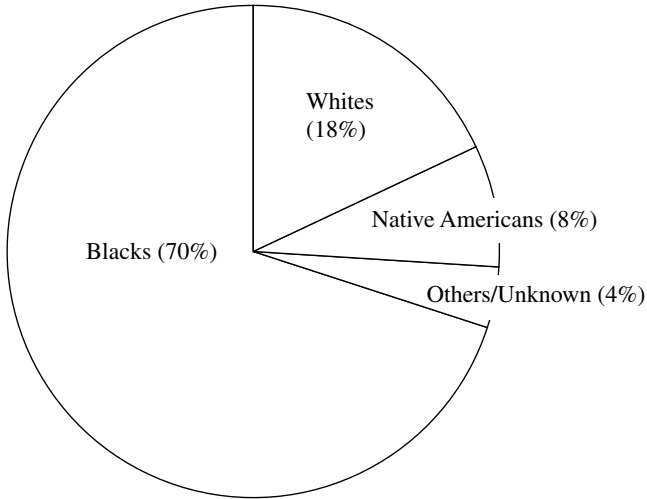


FIGURE 1.3 Children living in crack households.

TABLE 1.4

Cause of death	Number of deaths
Heart disease	12 378
Cancer	6448
Cerebrovascular disease	3958
Accidents	1814
Others	8088
Total	32 686

Example 1.8

Table 1.4 lists the number of deaths due to a variety of causes among Minnesota residents for the year 1975. After calculating the proportion of deaths due to each cause: for example,

$$\begin{aligned} \text{deaths due to cancer} &= \frac{6448}{32\ 686} \\ &= 0.197 \text{ or } 19.7\% \end{aligned}$$

we can present the results as in the pie chart shown in Figure 1.4.

Line Graphs A line graph is similar to a bar chart, but the horizontal axis represents time. In the applications most suitable to use line graphs, one binary characteristic is observed repeatedly over time. Different “groups” are consecutive years, so that a line graph is suitable to illustrate how certain proportions change over time. In a line graph, the proportion associated with each year is represented by a point at the appropriate height; the points are then connected by straight lines.